BED EFFICIENCY FOR DISPLACEMENT WASHING OF KRAFT SOFTWOOD AND HARDWOOD PULPS

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Received July 19, 2017

Using a laboratory displacement washing cell, kraft long fibre softwood and short fibre hardwood pulps cooked from spruce and a blend of deciduous woods, respectively, were subjected to displacement washing with water as a wash liquid. Based on the washing breakthrough curves measured for alkali lignin, the Péclet number was evaluated and the washing efficiency was characterised by bed efficiency at a wash liquor ratio equal to unity. For both pulps, bed efficiency increases with increasing Péclet number, however, it decreases with increasing dimensionless dispersion length. At pulp bed consistency of 120 to 130 kg m^{-3}, the bed efficiency of short fibre hardwood pulp was found to be greater in comparison with that of long fibre spruce pulp, which has lower permeability and greater specific hydraulic resistance. Since the displacement was the dominant mechanism during the initial period of the washing process, bed efficiency greater than unity was achieved for both pulps.

Keywords: pulp, spruce, hardwood, displacement washing, bed efficiency, dispersion, Péclet number

INTRODUCTION

Pulp washing is one of the main unit operations in chemical pulping and bleaching of pulp. Commercial pulp washers are based on dilution, followed by thickening and/or displacement washing mechanisms.[1] In displacement washing, the wash water or weak liquor is forced through the pulp bed, displacing mother liquor from it.

In order to characterize non-ideal flow patterns within the packed bed of non-porous particles, the dispersion flow model containing only one dimensionless parameter, the Péclet number, is widely used.[2–9] This model describing the flow of the fluid phase through porous media can be also applied for displacement washing of pulp.[10–14]

Many studies have been devoted to investigating the effect of process variables, such as dilution factor,[15,16] pulp bed consistency, pulp bed thickness, wash liquid velocity, and temperature of wash liquid[17–19] on the washing efficiency. Since the pulp bed formed from randomly oriented fibres with central cavity, lumen, is a complex system influencing the flow of the wash liquid inside the bed and, moreover, the displacement washing of pulp fibres is influenced by a number of phenomena occurring in porous media (geometrical properties of fibres, including their swelling, intrinsic properties of miscible fluids, sorption, electrokinetic phenomena and others), the results obtained seem to be somewhat inconsistent.

In this paper, the efficiency of the displacement washing of kraft spruce and hardwood pulps was characterised by bed efficiency, which is analogous to the Murphree efficiency used commonly for absorption and distillation plates. Hydraulic pulp bed properties were described by the equivalent pore diameter, specific resistance and permeability coefficient.

THEORETICAL BACKGROUND

The quality of the displacement washing of pulp is often expressed by the traditional wash yield, $WY_{RW=1}$, expressed as:[13,14,17,19]

$$WY_{RW=1} = \frac{\int_{RW=0}^{RW=1} \frac{\rho_e}{\rho_0} \mathrm{d}(RW)}{\int_{RW=0}^{RW\to\infty} \frac{\rho_e}{\rho_0} \mathrm{d}(RW)} \qquad (1)$$

which can be defined as the mass of solute washed out at the wash liquor ratio equal to unity divided by the total mass of solute removed from the pulp bed during the washing process. The meaning of the symbols in Equation (1) is as follows (*cf*. Fig. 1): ρ_e is the solute concentration in the outlet stream, ρ_0 is the initial solute concentration in the bed at the start of displacement, and *RW* is the wash liquor ratio directly proportional to time and defined as the ratio of the amount of wash liquid passed through the bed and the initial amount of mother liquor present in the bed.

The efficiency of a packed bed may be also defined by the relationship similar to the Murphree efficiency,[20] which is often used for distillation trays, as:

$$E = \frac{\rho_e - \rho_i}{\rho_{avg} - \rho_i} \tag{2}$$

where ρ_i is the solute concentration in the wash liquor and ρ_{avg} is the average solute concentration inside the bed (*cf*. Fig. 1). The bed efficiency is the change in wash stream composition divided by the change that would have occurred if the outlet liquor composition was the same as the average liquor in the pulp bed.

For cross-flow washing mode, when the wash water enters the pulp mat moving transversely through the washing zone, Cullinan[21] derived the relationship between the efficiency of a pulp bed, *E*, the wash liquor ratio, *RW*, and the displacement ratio, *DR*, as follows:

$$DR = 1 - \exp(E \; RW) \tag{3}$$

In the case of an unmovable pulp bed, the displacement ratio may be defined as:

$$DR = \frac{\rho_0 - \rho_{avg}}{\rho_0 - \rho_i} \tag{4}$$

If the solute concentration in the wash liquid $\rho_i = 0$ and the initial and final pulp consistencies are the same, that is, the liquor volume associated in the pulp bed has a constant value, the displacement ratio at $RW = 1$ is equal to the wash yield defined by Equation (1). Then, the bed efficiency can be expressed as:

$$E = -\frac{\ln(1 - DR)}{RW} \tag{5}$$

The bed efficiency evaluated at the wash liquor ratio equal to unity, $E_{RW=1}$, was used to compare the lignin removal during the displacement washing process.

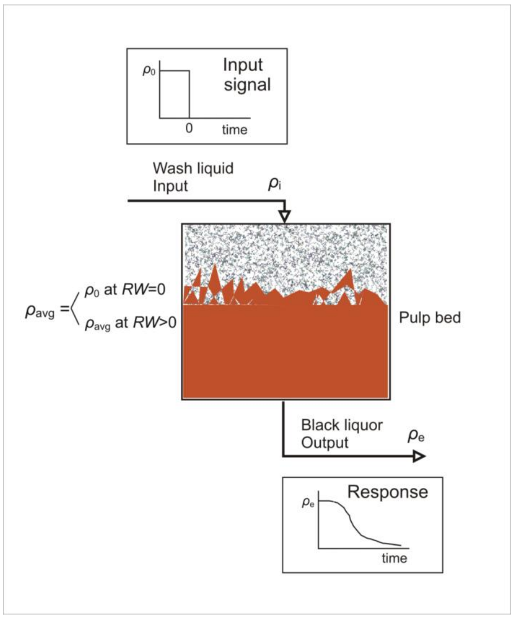

Figure 1: Illustration of displacement washing

The response to a step change in concentration provides time dependences called washing or also breakthrough curves. The shape of the washing curve can be characterised in terms of the dimensionless Péclet number based on the bed thickness[14] and derived from the mass balance of the tracer,[22] in our case, alkali lignin, for a given system in unsteady state in the following form:

$$Pe = \frac{h\,u}{D\,\varepsilon} \qquad (6)$$

where h is the thickness of the pulp bed, u is the wash liquid superficial velocity, D is the longitudinal dispersion coefficient, and ε is the average effective porosity[23] of a packed bed.

The Péclet number indicates the level of dispersion in the packed bed. Physically this means that, when the Péclet number approaches zero, the bed behaves like a perfect mixing vessel. On the other hand, the perfect plug flow is characterised by the Péclet number approaching infinity. The Péclet number also signifies the ratio of the convective to the diffusive transport mechanisms.

Mauret and Renaud[24] reported that pore size distribution or wall effects are parameters inducing anomalous dispersion, particularly in consolidated porous media. In order to quantify a measure of dispersion in the packed bed, the dispersion length $l_d = h/Pe$ seems to be a suitable parameter. Assuming a cylindrical shape of pulp fibres, when the pulp fibre length is much greater than fibre diameter, then the dimensionless dispersion length, $L_D = l_d/d_f$, for the fibre diameter $d_f \approx 4/a_v$, can be expressed as:[24]

$$L_D = \frac{h\,a_v}{4\,Pe} \qquad (7)$$

where a_v is the specific surface of pulp fibres.

EXPERIMENTAL

Kraft pulp cooking

Spruce wood mill chips without undesirable components, such as bark, oversized chips and knots, were stored under laboratory conditions for 2 years and used for kraft pulping. The chips were classified according to their length, width and thickness within the following limits: 20 mm < length < 59 mm, 10 mm < width < 30 mm, 1.6 mm < thickness < 8.4 mm. Their moisture content was of approximately 6%. Batch kraft cooking was carried out in a laboratory rotary digester, comprising six autoclaves of 750 cm^3 capacity, immersed in an oil bath. The cooking conditions, when the rejects amount was acceptable, were as follows: liquor-to-wood ratio of 4:1, active alkali charge of 18 mass % expressed as Na_2O per oven-dried wood, and cooking temperature of 168 °C. Industrial white liquor, at a sulphidity of 28%, was used. The temperature regime was as follows: 45 min heating to 110 °C, 45 min dwelling at this temperature, 60 min heating to 168 °C, and then dwelling at cooking temperature to achieve the desired H-factor of 1020 h. The kraft pulp was disintegrated with a laboratory Lorentzen & Wettre disintegrator, washed up with tap water by dilution/thickening, screened manually using a 10 mesh sieve, dewatered to 30% consistency, using a laboratory centrifuge machine, and stored in a refrigerator at 8 °C. The kappa number of 32.3 was determined according to Tappi Test Method T 236 om-99. Using a Kajaani instrument, the average lengths of fibres in the wet state, as well as the fibre coarseness were determined (Table 1).

The kraft hardwood pulp was cooked from a blend of deciduous trees, namely beech 55%, oak 16%, Turkey oak 8%, acacia 4%, hornbeam 5%, ash 1%, alder 2%, aspen 2%, poplar 6%, and others 1%. Cooking conditions, as well as geometrical fibre properties, can be found in our previous paper.[25]

Table 1
Geometrical characteristics of pulp fibres

Pulp	Arithmetic average length, mm	Weighted average length, mm	Polydispersity of fibre length	Fibre coarseness, mg m^{-1}	Specific surface	
					cm^{-1}	$m^2\,kg^{-1}$
Spruce	1.88	2.68	1.43	0.123	2360	853
Hardwood	0.749	0.899	1.20	0.370	3380	983

Displacement washing

Displacement washing experiments simulated under laboratory conditions were performed in a cylindrical glass cell with the inside diameter of 35 mm under constant pulp bed height of 30 mm. Pulp beds were formed from a dilute suspension of unbeaten unbleached kraft pulp in black liquor. In the case of the spruce pulp, the properties of black liquor were as follows: solids content of 19.5 mass % (of which ash represented 64.3% and organic substances – 35.7%), density of 1097 kg m^{-3} at 22 °C, pH value of 12 and alkali lignin concentration of 56 g dm^{-3}. The properties of black liquor displaced from hardwood pulp bed were reported in our previous paper.[25] After compression to the desired thickness of 30 mm, the consistency, *i.e.*, mass concentration of moisture-free pulp fibres in the bed varied within the limits from 122 to 129 kg m^{-3}. The pulp beds were not mechanically conditioned and were used as prepared.

To investigate the displacement washing process, the stimulus-response method was chosen. Distilled water at the temperature of 22 °C employed as a wash liquid was distributed uniformly through the piston to the top of the bed at the start of the washing experiment, approximating a step change in the alkali lignin concentration. At the same time, the displaced liquor was collected at the atmospheric pressure from the bottom of the bed through the septum. The washing effluent was sampled at different time intervals until the effluent was colourless. Samples of the washing effluent leaving the pulp bed were analysed for alkali lignin using an ultraviolet spectrophotometer Cintra 10e, operating at the wavelength of 295 nm. Displacement washing experiments with pulp fibres, as well as the washing equipment, were described in detail in a preceding paper.[14]

After completing the washing run, the volumetric flow rate of the wash liquid was measured gravimetrically at the pressure drop of 7 kPa to determine the permeability and average porosity of the pulp bed. Analogous measurements at various consistencies of the bed were focused on determining the effective specific surface of pulp fibres according to Ingmanson.[26]

RESULTS AND DISCUSSION

First of all, it must be emphasized that the results obtained for kraft spruce pulp are compared with those obtained for kraft hardwood pulp cooked to the kappa number of 29.9 earlier.[25] In the preceding paper,[25] only the traditional wash yield, axial dispersion coefficient and time parameters were reported. Of course, on the basis of the breakthrough curves measured for short fibre hardwood pulp, the further washing and fibre bed characteristics, such as the bed efficiency, dispersion length, equivalent pore diameter, specific bed resistance and permeability coefficient, were evaluated and used for comparison with the long fibre spruce pulp in the present paper. All displacement washing experiments were performed at the dimensionless length ratio $h/d = 0.86$.

Bed efficiency

The bed efficiency defined by Equation (2) was evaluated at the wash liquor ratio equal to unity. In the case of pure displacement, when a perfect plug flow exists, the efficiency increases from 1 to infinity with increasing wash liquor ratio from 0 to 1. For a perfectly mixed vessel, when the outlet lignin concentration is equal to the average concentration inside the bed, the efficiency is equal to unity.

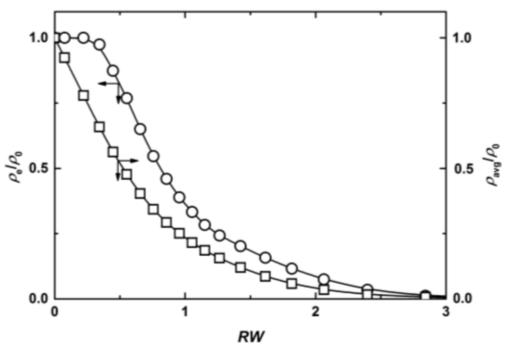

Figure 2: Typical breakthrough curve, ρ_e/ρ_0 *vs.* RW (○), and normalised average alkali lignin concentration in the pulp bed, ρ_{avg}/ρ_0 (□), for displacement washing of kraft spruce pulp

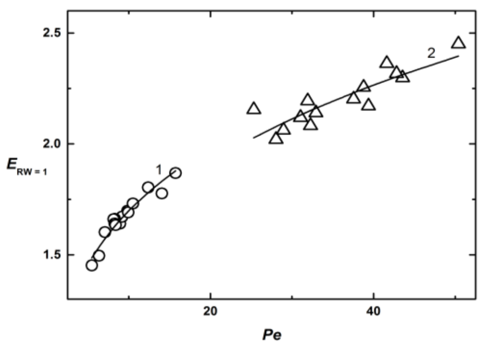

Figure 3: Bed efficiency at $RW = 1$ as a function of Péclet number for spruce (○) and hardwood (△) pulps (solid lines: 1 Eq. (8), 2 Eq. (9))

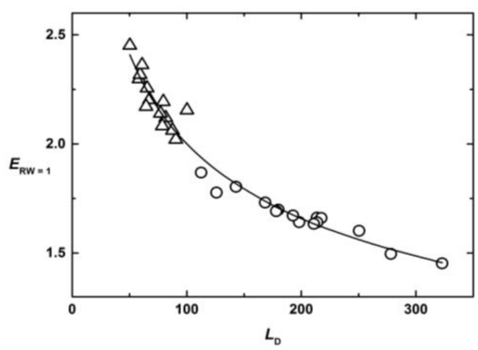

Figure 4: Bed efficiency at $RW = 1$ as a function of dimensionless dispersion length for spruce (○) and hardwood (△) pulps (solid line Eq. (10))

In displacement washing of pulp fibres, when the exit solute concentration is greater than the average solute concentration inside the bed, bed efficiency greater than unity should be measured.

It is illustrated by Figure 2, where the typical breakthrough curve along with the dimensionless average concentration of alkali lignin inside the pulp bed measured for the displacement washing of kraft spruce pulp are shown.

For kraft spruce and hardwood pulps, the bed efficiency at the wash liquor ratio equal to unity, $E_{RW=1}$, calculated from Equation (5) is plotted against the Péclet number in Figure 3. As expected, the bed efficiency greater than unity increases with increasing Péclet number. For the spruce pulp, the following correlation equation:

$$E_{RW=1} = 1.02\ Pe^{0.223} \tag{8}$$

with a mean relative deviation of 1.2%, 95% confidence limits of the coefficient (0.997; 1.04) and of the power of the Péclet number (0.213; 0.232), was derived in the Péclet number range from 5.5 to 15.7.

For hardwood pulp, the following correlation:

$$E_{RW=1} = 0.927\ Pe^{0.242} \tag{9}$$

with a mean relative deviation of 1.8%, 95% confidence limits of the coefficient (0.855; 0.997) and of the power of the Péclet number (0.219; 0.265), was derived in the Péclet number range from 25.3 to 50.4.

The relatively great values of the bed efficiency obtained mainly for hardwood pulp confirm that the displacement prevails over leaching in the initial washing period for $RW \leq 1$. The influence of the Péclet number on the bed efficiency was similar to that for the displacement of black liquor from the packed bed of 1.9 mm glass beads when the relation $E_{RW=1} = 1.05\ Pe^{0.23}$ was found in the Péclet number range from 33 to 82.[14] However, for non-porous glass spheres, when dispersion at the interface between the wash liquid and the entrained liquor exists, the bed efficiency was greater, from 2.22 to 2.85, depending upon the Péclet number,[14] than that for the pulp bed comprising randomly oriented porous fibres from which a solute is transported by diffusion into the wash liquid, so that leaching accompanied the displacement mechanism.

With respect to the character of the unmovable pulp fibre bed consisting of compressible porous fibres, where geometrical similarity does not exist, the influence of the dimensionless dispersion length on the bed efficiency is shown in Figure 4. The values of the dimensionless dispersion length ranging from 58 to 322 confirm that pulp fibre beds rank with dispersive media characterised by the dimensionless dispersion length from 4 to 300,[24] while for a packed bed of 1.9 mm glass beads, the dimensionless dispersion length was between 0.2 and 0.5 only.[14]

The relation between the bed efficiency and dimensionless dispersion length may be expressed in the following form:

$$E_{RW=1} = 6.96\ L_D^{-0.271} \tag{10}$$

with a mean relative deviation of 2.4%, 95% confidence limits of the coefficient (6.85; 7.07) and of the power of the Péclet number (–0.274; –0.267) for both kraft pulps, spruce and hardwood, tested in this work.

Axial dispersion coefficient

The dispersion of alkali lignin in the pulp bed is induced particularly by mechanical or geometric dispersion resulting from the fluctuations of the local wash liquid velocity caused by fibres random configuration and also by molecular diffusion at the interface between the miscible fluids, namely wash liquid and black liquor.[24] Moreover, the molecular diffusion is the basic mechanism for leaching of alkali lignin from within fibre walls into the wash liquid, flowing through pores particularly in the last washing period, recorded as a small breakthrough curve tailing for RW above 2.5 approximately.

From Figure 5, it follows that the dependence of the axial dispersion coefficient calculated from Equation (6) on the superficial wash liquid velocity shows an increasing trend for both kraft pulps, in accordance with previously reported results.[14] However, the values of the dispersion coefficient show a large scattering, particularly for spruce pulp fibres, in contrast to packed beds formed by non-porous particles, such as glass spheres,[8,14] Raschig rings and nickel foams,[8] where a linear dependence between the axial dispersion coefficient and wash liquid velocity was found. Similarly, Sherman[6] reports that the mixing parameter, D/u, was a constant for packed beds built up by either synthetic fibres or 3 mm glass spheres.

In agreement with Mauret and Renaud,[24] the results obtained showed that the pulp fibre bed formation is a critical step influencing alkali lignin dispersion during displacement washing. Even if the experimental conditions during pulp bed formation are strictly identical, the difference in geometry, that is, in average pore size and in pore size distribution must be taken into account. Nevertheless, the axial dispersion coefficient calculated for both types of pulp was plotted against the dimensionless dispersion length in Figure 6. In spite of the scattering in the data, the dependence of the dispersion coefficient on the dispersion length may be approximately expressed by the linear relation:

$$D = 4.66 \times 10^{-9} \; L_D \qquad (11)$$

with a correlation coefficient of 0.92. As expected the axial dispersion coefficient increases with decreasing Péclet number. These results confirm that the higher axial dispersion coefficient achieved for spruce pulp fibre beds had a negative impact on the bed efficiency, which was lower compared with the hardwood pulp.

Pulp fibre bed characteristics

In displacement washing of pulp, the wash liquid flows through the bed of compressible porous particles with the central cavity called lumen. It is not clear so far whether the wash liquid flows through lumens or not. In spite of this fact, the resistance to the flow of a wash liquid through the voids in the bed of pulp fibres results from the total drag of all the fibres in the bed.[20] With respect to a low Reynolds number (defined in Symbols), Re < 0.01, the laminar flow regime may be assumed. In this case, the wash liquid flow is directly proportional to the pressure drop and inversely proportional to the wash liquid viscosity, that is, Darcy's law describes the flow of wash liquid.

Besides the total hydraulic bed resistance, the specific resistance of pulp fibre beds defined as:

$$\alpha = \frac{(1-\varepsilon)^2 \; a_V^2 \; K}{\varepsilon^3 \; \rho_F} \qquad (12)$$

which is, for incompressible beds, independent of the pressure drop and of the position in the bed, can be applied as a hydraulic parameter. The average specific bed resistance is a measure of the resistance offered by the fibre bed to the flow of the wash liquid.

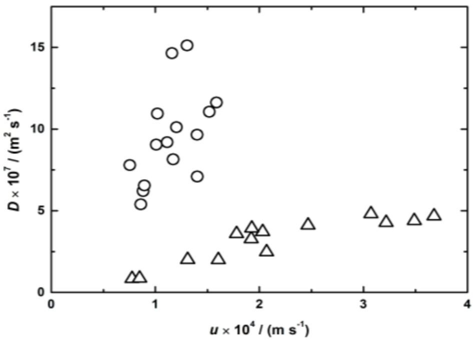

Figure 5: Influence of superficial wash liquid velocity on axial dispersion coefficient for spruce (○) and hardwood (△) (ref.[25]) pulps

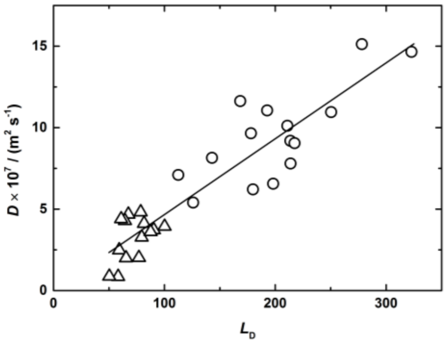

Figure 6: Influence of normalised dispersion length on axial dispersion coefficient for spruce (○) and hardwood (△) pulps (solid line Eq. (11))

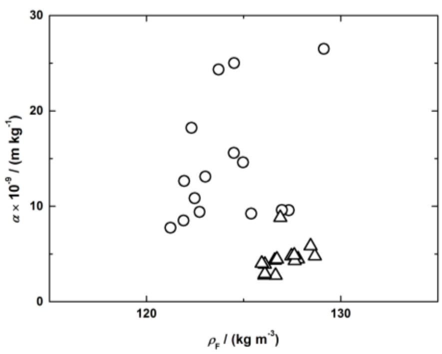

Figure 7: Specific bed resistance as a function of bed consistency for spruce (○) and hardwood (△) pulps

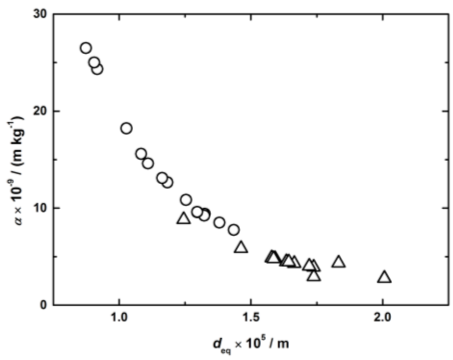

Figure 8: Specific bed resistance as a function of equivalent pore diameter for spruce (○) and hardwood (△) pulps

Equation (12) shows that the specific bed resistance is influenced solely by the physical properties of the bed, especially the fibre size and the porosity.[20] It follows from Figure 7 that, although the bed consistency of spruce pulp is slightly lower, the specific bed resistance of spruce pulp is unambiguously greater in comparison with that of hardwood pulp. The reason is that the average effective porosity directly proportional to the void volume in the bed[23] varied within the ranges from 0.34 to 0.45 and from 0.51 to 0.63 for spruce pulp and hardwood pulp, respectively.

The bundles of pulp fibres are deposited from the pulp slurry to form a complicated network of pores inside the pulp bed. Then, the fibre bundles influence the bed resistance rather than the geometry of individual fibres. Owing to the pore size variety, the equivalent pore diameter defined as 4 times the hydraulic radius given as the ratio of the cross-sectional area of the pore to the wetted perimeter of the pore may be expressed as:

$$d_{eq} = \frac{4\,\varepsilon}{(1-\varepsilon)\,a_V} \tag{13}$$

to characterise closely the pore size inside the pulp bed. Figure 8 illustrates that the specific bed resistance decreases with increasing equivalent pore diameter for both pulps investigated in this work. Moreover, lower values of the specific resistance for hardwood pulp follow smoothly the specific resistance values determined for spruce pulp.

One can assume that the greater the mass of black liquor occupying the void volume of the pulp bed, the lower the specific bed resistance is. In Figure 9, the dependence of the specific bed resistance upon the mass fraction, L_ε/L_0, where L_ε is the mass of black liquor in the void spaces of the pulp bed, directly proportional to the effective porosity, and L_0 is the total mass of black liquor originally present in the pulp bed, is illustrated. Comparing Figures 8 and 9, it is evident that the dependences α *vs*. d_{eq} and α *vs*. L_ε/L_0, have a similar trend. At the same time, it also seems that the washing efficiency for the pulp, such as hardwood pulp, where a relatively greater amount of black liquor available for displacement mechanism is present in the inter-particle voids, is greater than that for spruce pulp, which has a lower mass fraction L_ε/L_0. Of course, it is worth mentioning that both systems were different in the initial black liquor concentration expressed for alkali lignin, which was 56 kg m^{-3} and 27 kg m^{-3} for spruce and hardwood pulps, respectively. As reported earlier,[14,17] the effect of the initial black liquor concentration on the washing efficiency should be taken into account as well.

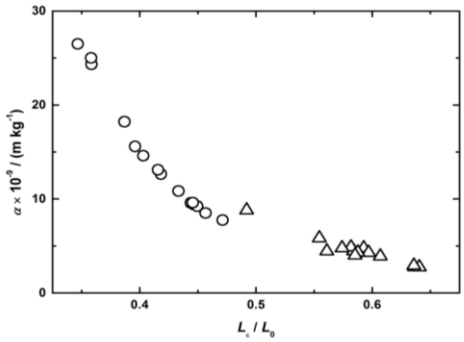

Figure 9: Specific bed resistance as a function of mass fraction of black liquor in void volume of pulp bed for spruce (○) and hardwood (△) pulps

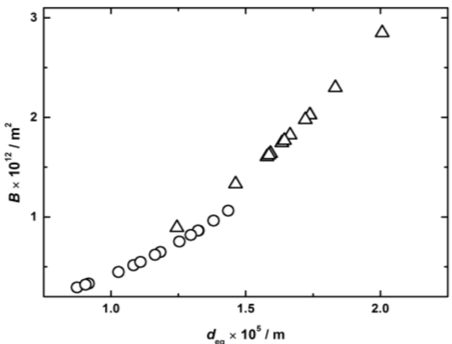

Figure 10: Bed permeability as a function of equivalent pore diameter for spruce (○) and hardwood (△) pulps

The permeability of the pulp fibre bed describes how easily a wash liquid is able to move through the porous bed. Hence, the permeability coefficient depends on the combination of the fluid and porous material properties.[20] For a constant pulp bed thickness and a laminar flow regime, when Darcy's law holds, the permeability is directly proportional to wash liquid velocity and viscosity and inversely proportional to the driving force, *i.e*., pressure drop. The dependence of the pulp bed permeability on the equivalent pore diameter for spruce and hardwood pulps is illustrated in Figure 10. As expected, in contrast to the specific bed resistance (*cf*. Fig. 8), the bed permeability increases with increasing the equivalent pore diameter. Hence, greater bed permeability was reached for hardwood pulp in comparison with spruce pulp. It should be noted that, in the consistency range studied in the present work, the permeability properties measured for spruce and hardwood pulps are comparable with those reported by Rainey *et al*.[27] for bagasse pulp.

CONCLUSION

The results obtained from the displacement washing experiments carried out in our laboratory showed that the washing efficiency characterised by the bed efficiency at the unit wash liquor ratio is unambiguously lower for long fibre spruce pulp, having greater specific bed resistance and lower equivalent pore diameter, in comparison with short fibre hardwood pulp cooked from a blend of deciduous woods, in which a greater fraction of black liquor was associated in void spaces and available for the displacement mechanism. The bed efficiency increases with increasing Péclet number for both long fibre spruce and short fibre hardwood pulps, however, it decreases with increasing dispersion length proportionally to the reciprocal of the Péclet number. Furthermore, the dependence of the axial dispersion coefficient on the dimensionless dispersion length shows a linear trend. On the other hand, the lower bed efficiency measured for spruce pulp can be also influenced by the high initial alkali lignin concentration of 56 kg m^{-3}, in contrast to 27 kg m^{-3} in the case of hardwood pulp. Nevertheless, the results obtained in the present work showed that the bed efficiency is a suitable tool for describing the washing efficiency of pulp and justify further work in the area of displacement washing analysis.

ACKNOWLEDGEMENT: This work was supported by the Internal Grant Agency of the University of Pardubice under the research project SGSFCHT_2017_006.

SYMBOLS

a_v specific surface of fibres based on volume, m^{-1}
B permeability, m^2
D dispersion coefficient, $m^2\ s^{-1}$
DR displacement ratio defined by Eq. (4)
d inner diameter of washing cell, m
d_{eq} equivalent diameter of pores defined by Eq. (13), m
E bed efficiency defined by Eq. (2)
$E_{RW=1}$ bed efficiency at $RW = 1$
h bed thickness, m
K Kozeny constant (dimensionless) in Eq. (12)
L_D dimensionless dispersion length defined by Eq. (7)
L_0 mass of black liquor originally present in the pulp bed
L_ε mass of black liquor in the void spaces of the pulp bed
ΔP pressure difference, Pa
Pe Péclet number based on the thickness of the bed and defined by Eq. (6)
Re Reynolds number ($= 4u\rho_{WL}/(a_v(1-\varepsilon)\mu)$)
RW wash liquor ratio
u superficial wash liquid velocity, m s^{-1}
$WY_{RW=1}$ wash yield at $RW = 1$ defined by Eq. (1)

Greek letters

α average specific bed resistance defined by Eq. (12), m kg^{-1}
ε average bed porosity
μ liquid viscosity, Pa s
ρ_{avg} average lignin concentration inside the pulp bed, kg m^{-3}
ρ_e exit lignin concentration, kg m^{-3}
ρ_F consistency of pulp, kg m^{-3}
ρ_i lignin concentration in the wash liquid, kg m^{-3}
ρ_{WL} density of wash liquid, kg m^{-3}
ρ_0 initial lignin concentration in the pulp bed, kg m^{-3}

REFERENCES

[1] R. H. Crotogino, N. A. Poirier and D. T. Trinh, *Tappi J*., **70**, 95 (1987).
[2] O. Levenspiel and W. K. Smith, *Chem*. *Eng*. *Sci*., **6**, 227 (1957).
[3] E. T. van der Laan, *Chem*. *Eng*. *Sci*., **7**, 187 (1957).
[4] K. B. Bischoff and O. Levenspiel, *Chem*. *Eng*. *Sci*., **17**, 245 (1962).
[5] H. Brenner, *Chem*. *Eng*. *Sci*., **17**, 229 (1962).
[6] W. R. Sherman, *AIChE J*., **10**, 855 (1962).
[7] G. L. Pellett, *Tappi J*., **49**, 75 (1966).
[8] A. Montillet, J. Comiti and J. Legrand, *Chem*. *Eng*. *J*., **52**, 63 (1993).
[9] A. Leitão, P. Carlos, S. Santos and A. Rodrigues, *Chem*. *Eng*. *J*., **53**, 193 (1994).
[10] L.-E. Grähs, *Svensk Papperstidn*., **78**, 446 (1975).
[11] L.-E. Grähs, *Svensk Papperstidn*., **79**, 84 (1976).
[12] L.-E. Grähs, *Svensk Papperstidn*., **79**, 123 (1976).
[13] N. A. Poirier, R. H. Crotogino and W. J. M. Douglas, *Can*. *J*. *Chem*. *Eng*., **66**, 936 (1988).
[14] F. Potůček, *Collect*. *Czech*. *Chem*. *Commun*., **62**, 626 (1996).
[15] P. F. Lee, *Tappi J*., **62**, 75 (1979).
[16] P. F. Lee, *Tappi J*., **67**, 100 (1984).
[17] D. T. Trinh, N. A. Poirier, R. H. Crotogino and W. J. M. Douglas, *J*. *Pulp Paper Sci*., **15**, J28 (1989).
[18] R. J. Smith and G. D. Geoffrey, in *Procs*. *53rd Appita Annual General Conference*, vol. 1, 1999, pp. 275-282.
[19] N. A. Poirier, R. H. Crotogino, D. T. Trinh and W. J. M. Douglas, in *Procs*. *Pulp Washing Symposium*, Finnish Pulp and Paper Research Institute, Finland, May 18-21, 1987.

[20] W. L. McCabe, J. C. Smith and P. Harriott, "Unit Operations of Chemical Engineering", 6th ed., McGraw-Hill, Boston, 2001.
[21] H. T. Cullinan, *Appita J*., **44**, 91 (1991).
[22] F. Potůček, *Pap*. *Celul*., **56**, 8 (2001).
[23] J. D. Lindsay, *Tappi J*., **77**, 225 (1994).
[24] E. Mauret and M. Renaud, *Appita J*., **55**, 123 (2002).
[25] F. Potůček and J. Miklík, *Chem*. *Pap*., **64**, 147 (2010).
[26] W. L. Ingmanson, *Chem*. *Eng*. *Prog*., **49**, 577 (1953).
[27] T. J. Rainey, W. O. S. Doherty, D. M. Martinez, R. J. Brown and N. A. Kelson, *Appita J*., **62**, 296 (2009).